Engineering Notes

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Inspection Frequency Optimization for Aircraft Structures Based on Reliability Analysis

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Nomenclature

C_I = cost of one inspection for one airplane
C_f = cost of failure of one airplane
C^* = total expected cost given by Eq. (1)
C_r = nondimensional relative expected cost
M = total number of airplanes in a fleet, called "fleet size"
N = total number of periodic inspections for each airplane in the design service lift
Pa = a prescribed (or specified) allowable level of failure probability for each airplane in its service life, i.e., the failure probability of each airplane should be less than or equal to Pa
P_f = probability of first failure of a fleet of M airplanes in the design service life
P_f^* = a prescribed (or specified) allowable level of first failure probability for a fleet of airplanes
γ = ratio of the cost of one inspection for one airplane to the cost of failing one airplane C_I/C_f

I. Introduction

An exploratory reliability analysis of aircraft structures has been presented in Ref. 1. It has been shown that inspection has a significant effect on the reliability of a fleet of aircraft. In particular, the fleet reliability increases as the inspection frequency or the inspection quality increases. However, the cost of inspection and maintenance increases also as the frequency or quality of the inspection increases. As a result, there is a tradeoff potential between the fleet reliability and the cost of inspection and maintenance. It is the purpose of this paper to present the results of an exploratory study in this area.

For spacecraft structures, the tradeoff between the structural reliability, the design safety factor, and the proof test level has been discussed in Refs. 2 and 3. In these papers, the concept of the cost of failure is employed to minimize the structural weight and to determine the optimum proof test level. The concept of the cost of failure is used herein to formulate an optimization schedule for the determination of the optimum inspection frequency for aircraft structures.

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II. Formulation

The reliability analysis of aircraft structures presented in Ref. 1 indicates that there are a number of variables which can be adjusted in such a way that an objective function of utility can be optimized. Some of these variables are, the inspection frequency, the inspection quality, the design stress level. For the sake of simplicity in presenting the basic idea, only the number of inspections will be considered as a variable, realizing that more variables can be treated in a similar fashion.

The objective function to be minimized is the "expected cost" while, at the same time, a prescribed level of fleet reliability is maintained. The expected cost C^* consists of the expected cost of inspection‡ and the expected cost of failure§ of airplanes,

$$C^* = NMC_I + C_f MP(N) \tag{1}$$

where N= total number of periodic inspections for each airplane in the design service life $[O,T]$; M= total number of airplanes in a fleet, called fleet size; C_I= cost of one inspection for one airplane, including the cost of repair of the cracked components when fatigue cracks are detected during inspection; $P(N)$= probability of failure of one airplane in the design service life $[O,T]$ under N periodic inspections; and C_f= cost of failure of one airplane.

In Eq.(1), $MP(N)$ is the average number of airplanes expected to fail during the design service life, since the number of failure in a fleet of M airplanes follows a binomial distribution. The first term in Eq. (1) denotes the expected cost of inspection and maintenance, and the second term the expected cost of failure.

Note that the first term in Eq. (1) is zero if no inspection is performed, i.e., N=0. The inspection and maintenance cost increases as N increases, but the cost of failure decreases since the probability of failure $P(N)$ decreases as the number of inspections N increases (see Ref. 1). A method for evaluating $P(N)$ has been discussed in Ref. 1 for transport-type aircraft. It is mentioned that both the cost of inspection C_I and the probability of failure $P(N)$ also depend on the quality of inspection. For simplicity in presentation, it is assumed that a specific method of inspection is employed.

The probability of first failure p_f in a fleet of M airplanes, which is related to $P(N)$,[1] should be less than a prescribed level of failure probability p_f^*.

$$p_f = 1 - [1 - P(N)]^M \leq p_f^* \tag{2}$$

The constraint of Eq. (2) can then be written as

$$P(N) \leq p_a \tag{3}$$

where $p_a = 1 - [1 - p_f^*]^{1/M}$ is a prescribed (or specified) allowable level of failure probability for one airplane. Therefore, the constraint on the fleet reliability is equivalent to the constraint on the reliability of each airplane. Equation (3) states that the probability of failure of an airplane $P(N)$

‡"Cost of inspection" includes cost of maintenance, the latter referring to minor repair such as reaming of cracked holes, patching or replacement of elements and minor components.

§"Cost of failure" to be interpreted as cost of the consequence of failure, which may be the cost of major damage; i.e., replacement of major component or whole aircraft, nonavailability of aircraft in service, etc.

should be less than a prescribed (or specified) allowable level of failure probability P_a.

Dividing Eq. (1) by MC_f, one obtains

$$C_r = \gamma N + P(N) \qquad (4)$$

where $C_r = C^*/MC_f$ is a nondimensional relative cost to be minimized, and

$$\gamma = C_I / C_f \qquad (5)$$

is the ratio of the cost of one inspection for one airplane to the cost of failing one airplane during the design service life. Hence, γ is the relative importance of the inspection cost compared to the cost of failure. It is an important parameter for the determination of the optimum inspection frequency N^*.

The optimum inspection frequency (the optimum number of inspections), N^*, is obtained by minimizing the relative cost C_r given by Eq. (4), when meanwhile the constraint of Eq. (3) should be satisfied. Since $P(N)$ is a nonlinear function of N,[1] the optimization is a nonlinear programing problem, and the mathematical techniques for obtaining the optimum solution are available in the literature.[4]

III. Numerical Example and Solution

The same numerical example given in Ref. 1 is considered. The relative cost C_r and the failure probability $P(N)$ are plotted as a function of N, the number of inspections (inspection frequency), for various values of γ in Fig. 1. It can be observed that each solid curve has a minimum. These minima, as connected by a dashed curve, may or may not be the solution for optimum inspection frequency depending on whether the constraint is active or inactive. When the equality sign in Eq. (3) (constraint) is satisfied by the optimum solution $N = N^*$, the constraint is said to be active. Otherwise, the constraint is inactive, in which case, the inequality sign holds. The procedure to obtain the optimum solution by graphical method is rather simple and is briefly described in the following.

When the point of intersection of the failure probability $P(N)$ and the constraint on the failure probability p_a in Fig. 1 is on the left of the minimum for a particular γ, the constraint is inactive and the minimum is the optimum solution. This is because, at the minimum, $P(N^*) < p_a$. For instance, if the constraint $p_a = 10^{-2}$, the intersection point is at A (see Fig. 1),

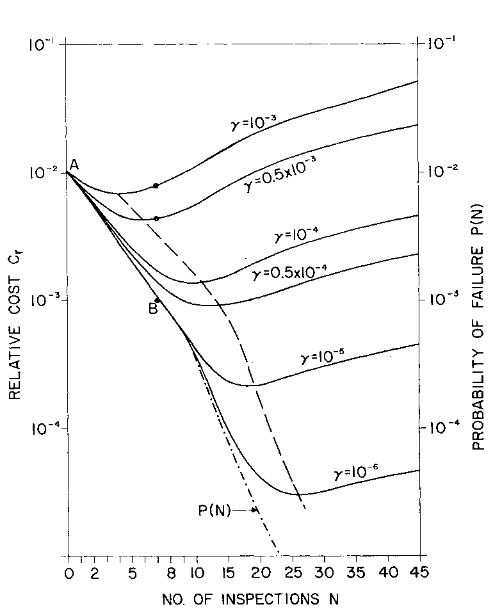

Fig. 1 Relative cost C_r and probability of failure $P(N)$ vs number of Inspections N; ——relative cost, ----probability of failure, ------ minimum relative cost.

and hence the dashed curve represents the true optimum solution. It can be observed from Fig. 1 that the present optimization approach not only minimizes the expected cost but also improves the probability of failure significantly [compare $P(N^*)$ and p_a]. In particular, a lower value of γ results in a higher optimum inspection frequency N^* as well as in a greater reliability improvement.

When the point of intersection of $P(N)$ and p_a is on the right of the minimum for a particular γ, the probability of failure $P(N)$ associated with the minimum of the solid curve is higher than the constraint p_a, and hence the minimum is not a feasible solution. Under this circumstance, the number of inspections associated with this intersection is the optimum solution. For instance, if $p_a = 10^{-3}$ the point of intersection of 10^{-3} abd $P(N)$ is denoted by B in Fig. 1. Hence for $\gamma < 0.3 \times 10^{-3}$, the minima lie on the right of B and hence they are the optimum solutions. For $\gamma > 0.3 \times 10^{-3}$, however, the minima are not the feasible solutions. The optimum solution is the inspection frequency associated with the point B as indicated by solid circles in Fig. 1.

A general characteristic observed from this study is that the smaller the γ value is, the higher the optimum number of inspections N^* will result. This characteristic is consistent with our intiution, because the smaller the inspection cost is compared to the cost of failure, the more frequent inspection can be afforded and hence higher fleet reliability can be achieved.

In the optimization process, it is not necessary to estimate the absolute value of both the cost of inspection C_I and the cost of failure C_f. All one has to estimate is γ, i.e., the relative importance of C_I to C_f. For instance, if the acquisition cost of the system is very high, (e.g. space shuttle), or if its failure has a serious consequence, such as loss of system and lives, γ will be very small and hence the optimum inspection frequency N^* is higher. Thus, higher improvement of the fleet reliability is achieved.

IV. Discussion

Only a concept of optimization for *inspection frequency* is formulated in this study for the sake of simplicity in presentation. Equation (4) can easily be extended to incorporate the inspection quality and the service stress level as variables. For instance, the inspection cost C_I is a function of the inspection quality, and the probability of failure of an aircraft $P(N)$ is a function of both the inspection quality and the service stress level (see Ref. 1). Then, the optimization procedure can be applied in a similar fashion to obtain the optimum inspection frequency, the optimum inspection quality and the optimum service stress level.

The present investigation is an attempt for the inspection frequency optimization of aircraft structures based on the concept of reliability analysis. Undoubtedly, there is a variety of other complicated problems associated with the inspection optimization. They may, however, be treated in an approach similar to that developed in this paper but with some modifications. It has been assumed that the cost of inspection and maintenance C_I can be lumped together as a first-order approximation. In many situations, however, the cost of maintenance, including the cost of repair or replacement, may be very significant and must be considered separately. It should then be estimated from the expected number of cracks and crack sizes, detected during each inspection, which in turn is a function of inspection frequency. Furthermore, for some types of military aircraft, the critical component is an integral structure in the airframe. In this case there is a small probability that the crack size is sufficiently large to make a replacement necessary. The replacement under this circumstance may mean the replacement of the entire wing. Therefore, the cost of replacement may be significantly higher than the cost of inspection and they should be considered as different variables. Furthermore, it has been indicated in Ref. 1 that the inspection at the later time of the service life is much more efficient. Consequently, it is possible to adjust or vary the lengths of the inspection intervals, e.g., longer inspection intervals in the early life time and shorter inspection intervals at the later service life, so that the maximum benefit can be achieved. The possibility of using or combining various inspection qualities or techniques to achieve either a maximum utility or a maximum improvement of fleet reliability can further be developed. Accordingly, the tradeoff between replacement, inspection quality, inspection interval, inspection frequency, retirement of aircraft, intended service life, etc., presents a broad spectrum of very interesting problems for further study.

V. Conclusion

An optimization scheme for the inspection frequency has been formulated on the basis of the expected-cost-of-failure concept. The optimum inspection frequency is determined by minimizing the expected cost while the constraint on the specified fleet reliability is satisfied. It has been shown that the optimum inspection frequency increases as the relative importance of the cost of inspection compared to the cost of failure becomes smaller, thus increasing the fleet reliability more significantly.

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Comparison of Sonic Boom Minimization Results in Real and Isothermal Atmospheres

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Nomenclature

a = speed of sound
A = equivalent area
CO = characteristic overpressure, $4I/T$
$F(y)$ = Whitham F function
I = impulse, $\int_{p>0} pdt$
L = equivalent length
M = Mach number
p = ambient pressure
Δp = overpressure
P = pressure perturbation
q = dynamic pressure
r = vertical distance from airplane axis
S = ray tube area
t = time, sec
T = total time between front and rear shock waves
W = cruise weight of airplane
x = axial distance
y = axial distance
α = advance
β = $(M^2-1)^{1/2}$
Γ = $(\gamma+1)/2$
ρ = density

Subscripts

h = altitude of initial waveform
g = ground level
L = equivalent length
y = axial distance

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FOR a cruising aircraft in an isothermal atmosphere, Seebass and George have provided a method which minimizes certain features of the pressure signature and yields the corresponding F function and equivalent area distribution, A.[1,2] To provide the same capability for a real atmosphere, their method has been modified using the appropriate equations of George and Plotkin[3] for horizontal advance,

$$\alpha_h = \frac{\Gamma M_h{}^3 F(y)}{(2\beta)^{1/2}} \int_o^r \frac{p_h}{p} \left(\frac{\rho a_h}{\rho_h a}\right)^{1/2} \left(\frac{S_h}{r_h S}\right)^{1/2} \frac{M}{\beta}\, dr$$

ray tube area,

$$\frac{S_h}{r_h S} = 1/M_h\left[1 - \frac{1}{M(r)^2}\right]^{1/2} \int_o^r \frac{dr}{[M(r)^2 - 1]^{1/2}}$$

and signature propagation

$$P_g = \left(\frac{S_h}{S_g}\right)^{1/2} \left(\frac{\rho_g a_g}{\rho_h a_h}\right)^{1/2} P_h$$

These results were programed on a digital computer and numerous calculations have been made for both atmospheres at varying flight conditions, using a scale height of 25,000 ft[1] for the isothermal atmosphere. The results shown in the figures herein are believed to be typical and are limited to pressure signatures in which the maximum overpressure has been minimized for the following conditions: altitude, 60,000 ft (18,288 m); weight, 600,000 lb (272,155 kgm); length, 300 ft (91.44 m); reflection factor, 2. Ratios of overpressure (Δp), impulse (I), and characteristic overpressure[4] *(CO)*, as predicted for the two atmospheres are shown in Fig. 1. For Mach numbers greater than 1.85, the isothermal approximation predicts overpressures given by the real atmosphere to within 1%. Predictions of impulse and characteristic overpressure are less accurate because of differences in signature length as shown for Mach 3 in Fig. 2, but for the same Mach range, these predictions fall within 5% of the real values. At lower supersonic Mach numbers the speed of sound gradient in the real atmosphere causes much more curvature of the ray tube than is predicted by the isothermal atmosphere,[3] thus larger errors occur for isothermal predictions in this Mach number range.

If the effects of aircraft wake and engine exhaust are neglected, and the aircraft volume is zero at its base, the

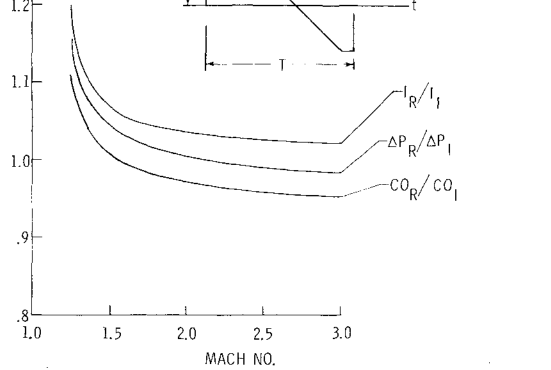

Fig. 1 Comparisons of signature parameters in real and isothermal atmospheres.